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* Formula for Polar Equation

Que 1. To find the radius of curvature for the Polar curve $r = f(\theta)$

or To Prove the formula : $\rho = \dfrac{(r^2+r_1^2)^{3/2}}{r^2+2r_1^2-rr_2}$ where the symbols have their usual meaning.

Proof :- We know that

$$\frac{1}{p^2} = \frac{1}{r^2} + \frac{1}{r^4}\left(\frac{dr}{d\theta}\right)^2 \quad \text{——— (1)}$$

Differentiating both sides with regard to θ, we get

$$-\frac{2}{p^3}\cdot\frac{dp}{d\theta} = -\frac{2}{r^3}\cdot\frac{dr}{d\theta} - \frac{4}{r^5}\left(\frac{dr}{d\theta}\right)^3 + \frac{2}{r^4}\cdot\frac{dr}{d\theta}\cdot\frac{d^2r}{d\theta^2}$$

or $$\frac{dp}{dr} = \left\{\frac{1}{r^3} + \frac{2}{r^5}\left(\frac{dr}{d\theta}\right)^2 - \frac{1}{r^4}\cdot\frac{d^2r}{d\theta^2}\right\}p^3$$

$$= \left\{r^2+2\left(\frac{dr}{d\theta}\right)^2 - r\cdot\frac{d^2r}{d\theta^2}\right\}\cdot\frac{p^3}{r^5} \quad \text{——— (2)}$$

From (1) $$\frac{1}{p^2} = \frac{r^2+\left(\frac{dr}{d\theta}\right)^2}{r^4} \quad \text{——— (3)}$$

or $$p = \frac{r^2}{\sqrt{r^2+\left(\frac{dr}{d\theta}\right)^2}}$$

We know that $\rho = r\dfrac{dr}{dp}$

$$= r\cdot\frac{r^5}{p^3\left\{r^2+2\left(\frac{dr}{d\theta}\right)^2 - r\frac{d^2r}{d\theta^2}\right\}} \quad \text{From (2)}$$

$$= \frac{r^6\left\{r^2+\left(\frac{dr}{d\theta}\right)^2\right\}^{3/2}}{r^6\left\{r^2+2\left(\frac{dr}{d\theta}\right)^2 - r\cdot\frac{d^2r}{d\theta^2}\right\}}$$

Hence,
$$\boxed{\rho = \frac{(r^2+r_1^2)^{3/2}}{r^2+2r_1^2-r\cdot r_2}}$$

Que ② :- To find the radius of curvature for the curve $u = f(\theta)$, where $u = \frac{1}{r}$.

We have.

$$u = \frac{1}{r} \quad \text{or} \quad r = \frac{1}{u} \qquad \text{——— (1)}$$

Diff. with respect to θ, we get

$$\frac{dr}{d\theta} = -\frac{1}{u^2} \cdot \frac{du}{d\theta} \qquad \text{——— (2)}$$

Again diff. with respect to θ, we get

$$\frac{d^2r}{d\theta^2} = \frac{u^2 \cdot \frac{d^2u}{d\theta^2} - \frac{du}{d\theta} \cdot 2u\frac{du}{d\theta}}{(u^2)^2}$$

$$= -\frac{u\frac{d^2u}{d\theta^2} - 2\left(\frac{du}{d\theta}\right)^2}{u^3} \qquad \text{——— (3)}$$

We know that

$$P = \frac{\left[r^2 + \left(\frac{dr}{d\theta}\right)^2\right]^{3/2}}{r^2 + 2\left(\frac{dr}{d\theta}\right)^2 - r\frac{d^2r}{d\theta^2}}$$

which, by virtue of (1), (2) and (3) is equivalent to

$$P = \frac{\left[\frac{1}{u^2} + \frac{1}{u^4}\cdot\left(\frac{du}{d\theta}\right)^2\right]^{3/2}}{\frac{1}{u^2} + 2\cdot\frac{1}{u^4}\left(\frac{du}{d\theta}\right)^2 + \frac{1}{u}\cdot\frac{u\frac{d^2u}{d\theta^2} - 2\left(\frac{du}{d\theta}\right)^2}{u^3}}$$

Hence $P = \dfrac{(u^2 + u_1^2)^{3/2}}{u^3(u + u_2)}$

where $u_1 = \dfrac{du}{d\theta}$

and $u_2 = \dfrac{d^2u}{d\theta^2}$